

Note: Attempt any FIVE questions, selecting at least ONE from each section.

Section I

Q.NO.01 (a) Discuss the continuity of $[x]$ for integral values of $x \in R$. (07)

(b) Evaluate $\int \frac{dx}{x^3 - 4x}$. (07)

(c) Find the equation of the tangent to the curve $x^2 - 4xy - 2y^2 + 4x - 2y + 31 = 0$ at $(1, 3)$. (06)

Q.NO.02 (a) Find $\lim_{x \rightarrow 0} \frac{\ln \sin^3 3x}{\ln \sin^4 2x}$. (07)

(b) If $Y(x) = \sin(m \sin^{-1} x)$ then prove that $(1-x^2)y''(x) - xy'(x) + m^2y(x) = 0$. (07)

(c) Expand $\sin x$ in power of x . (06)

Q.NO.3 (a) Evaluate $\int \frac{x^2}{(1+x^2)^3} dx$. (07)

(b) State and prove the Cauchy mean values theorem. (07)

(c) Show that $\int_0^{\frac{\pi}{2}} \ln \sin x dx = -\frac{\pi}{2} \ln 2$. (06)

Section II

Q.NO.4 (a) Find the tangent and normal to the circle $x = a \cos \theta, y = a \sin \theta$ at $p(a \cos \alpha, a \sin \alpha)$. (07)

(b) Find the pedal equation of the parabola $y^2 = 4a(x+a)$. (07)

(c) What does the equation $ax^2 + 2hxy + by^2 = 0$ represents if $h^2 < ab$. (06)

Q.NO.5 (a) Find the asymptotes of the curve $y^2 - x^2y - xy^2 + x^3 + x^2 - y^2 = 1$ (07)

(b) Find the area bounded by the curve $9y^2 + x = 2$ and the y -axis. (07)

(c) Find the curvature of $y^2 = 4ax$ at the point $(0, 0)$. (06)

Section III

Q.NO.6 (a) Test convergence or divergence $\sum_1^{\infty} n^2 e^{-3n}$ (07)

(b) Test convergence or divergence $\sum_1^{\infty} \frac{1}{9n^2 + 3n - 2}$. Also find its sum if possible. (07)

(c) Investigate the behavior of the series $\sum_1^{\infty} \frac{(n!)^2}{(n^2)!}$. (06)

Q.NO.7 (a) Expand in the fourier series $f(x) = |x|, -1 < x < 1$ (10)

(b) Find the fourier sine serie of $f(x) = \cos\left(\frac{\pi x}{p}\right), 0 \leq x \leq p$. (10)

Q.No.8

I. Define curvature and radius of curvature. (04)

II. Define fourier series. (04)

III. Define involute and evolute. (04)

IV. State Leibnitz Theorem. (04)

V. Evaluate $\int \sec^5 x dx$. (04)